

VIRTUAL ANALOG SIMULATION AND EXTENSIONS OF PLATE REVERBERATION

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ABSTRACT

In the 50s and 60s, steel plates were popularly used as a technique to add reverberation to sound. As a plate reverb itself is quite bulky and requires lots of maintenance, a digital implementation would be desirable. Currently, the available (digital) plugins rely solely on recorded impulse responses or simple delay networks. Virtual Analog (VA) simulations, on the other hand, rely on a model of the analog effect they are simulating, resulting in a sound and 'feel' the of the classical analog effect. In this paper, a VA simulation of plate reverberation is presented. Not only does this approach result in a very natural sounding reverb, it also poses many interesting opportunities that go beyond what is physically possible. Existing VA solutions, however, have limited control over dynamics of physical parameters. In this paper, we present a model where parameters like the positions of the in- and outputs and the dimensions of the plate can be changed while sound goes through. This results in a unique flanging and pitch bend effect, respectively, which has not yet been achieved by the current state of the art.

1. INTRODUCTION

A great number of digital audio effects is currently available to musicians and producers. Many sounds we could not even imagine a few years ago, we can now create using current DSP technology. However, despite this immense amount of options, there is still a great desire for the sound of the classical analog effects that made their first appearance in the late 40s and characterised music from the 50s and 60s onwards. Even though, generally, digital sound effects 'do the job', they do not have the 'feel' that the old analog effects had - something greatly desired by many musicians. Contrary to digital effect simulations, Virtual Analog (VA) simulations rely on a model of the analog effect they are simulating [1]. A great advantage of VA simulations over the original systems is that they do not age and thus do not require time consuming maintenance. Also, when digitalised they are easily accessible, mostly simpler to use and can be made much cheaper than their

analog counterparts. Naturally, the sound of a used analog audio effect can have its charm, but if desired, this can be modelled into the simulation. Also, VA simulations make it possible for parameters like room size, material properties, etc., to be changed, which is physically impossible or very hard to do. This can result in unique sounds that can only be created using VA simulations.

A popular reverberation technique used in the 50s and 60s was plate reverberation. A plate reverb utilises a small speaker (actuator) attached to a big steel plate to make it vibrate, and several pickups to pick up the sound after it has propagated through the plate. Several different plate reverbs made it to the market, the most popular being the EMT140 used in the Abbey Road Studios and undoubtedly leaving a mark on music in the aforementioned years. In fact, it was the only reverb used on Pink Floyd's *Dark Side of the Moon* [2]. A big issue of using an actual plate reverb is the sheer size and weight of it. The plate is 2x1m big and weighs (together with the rest of the installation) roughly 270kg [3], hence, a digital implementation of it would be desirable.

There are different ways of digitally implementing plate reverberation. Convolution is an effective approach, but not a VA one as it is not based on physical parameters. This means that flexibility is limited. Feedback Delay Networks (FDNs), as proposed by Jean-Marc Jot in [4], are also used as an implementation approach for creating a digital plate reverb by Jonathan Abel in [5]. FDNs are an efficient way of realising a VA solution to the plate reverberation problem. However, in his implementation, Abel uses a hybrid structure consisting of a convolutional part and an FDN-based part, making it not fully VA. Finite difference schemes as proposed in [6] and [7] are flexible, VA, but very computationally heavy solutions. Lastly, the vibrations of the plate can be decomposed into a series of plate modes - a modal description of the plate. It is both a VA and not computationally heavy approach and gives a lot of freedom in dynamic parameter manipulation.

Currently there are some VA plate reverb plugins available such as the *PA1 Dynamic Plate Reverb* [8] (that uses the aforementioned modal description) and the *Valhalla-Plate* [9]. These plugins, however, do not use the full potential of VA simulations. Parameters like for example pickup positions and sheet-size can be made dynamic, i.e., changed while sound is going through the plate. This will result in some interesting sounds and effects that can not be achieved with a physical plate reverb. In this paper we

propose a VA simulation of plate reverberation that utilises these dynamic parameters - something that the aforementioned plugins have not implemented. Furthermore, different kinds of damping that occur in physical plate reverbs are taken into account to make the simulation sound even more natural.

This work is structured as follows: in Section 2 the physics of a thin metal plate will be explained, in Section 3 a numerical solution will be described, in Section 4 the results of this solution will be discussed and lastly in Section 5 we will conclude and discuss future works for this project.

2. PHYSICS OF A THIN METAL PLATE

In order to simulate a plate reverb, a model of the physics of the plate is needed. The Kirchhoff-Love model mathematically describes stresses and deformations in thin plates subjected to external forces. The partial differential equation for an isotropic plate (including damping) is [10]:

$$\frac{\partial^2 u}{\partial t^2} = -\kappa^2 \nabla^4 u - c \frac{\partial u}{\partial t} + f(x_{\text{in}}, y_{\text{in}}, t). \quad (1)$$

Here, $u = u(x, y, t)$, a state variable that describes the transverse plate deflection and is defined for $x \in [0, L_x]$, $y \in [0, L_y]$ and $t \geq 0$, c is a loss parameter, ∇^4 is the bi-harmonic operator, $f(x_{\text{in}}, y_{\text{in}}, t)$ is the input signal at input source location $(x_{\text{in}}, y_{\text{in}})$ at time instance t and finally, κ^2 can be referred to as the stiffness parameter:

$$\kappa^2 = \frac{Eh^2}{12\rho(1-\nu^2)}, \quad (2)$$

where E , h , ρ , and ν are the Young's modulus, plate thickness, plate density and Poisson's ratio, respectively.

2.1 Frequency dispersion

One feature that characterises the sound of a plate reverb is the fact that higher frequencies propagate faster through a metal plate than lower frequencies [10, 11]. This phenomenon is called frequency dispersion. The dispersion relation is defined as:

$$\omega^2 \simeq \gamma^4 \kappa^2, \quad (3)$$

where ω is the angular frequency and γ the wavenumber. The group and phase velocities are defined as:

$$c_{\text{ph}} = \sqrt{\kappa\omega} \quad c_{\text{gr}} = 2\sqrt{\kappa\omega}. \quad (4)$$

Using the properties of the EMT140 (steel, 0.5mm thick) and a frequency range of 20-20,000kHz, group velocities can vary between ca. 20-620m/s. In regular (or room) reverberation all frequencies travel at the same speed making frequency dispersion one of the main differences between plate and room reverberation.

2.2 Damping

In plate reverbs, three different kinds of damping occur: thermoelastic damping, radiation damping and damping induced by a porous medium [10, 11]. All three will be shortly described in this section.

2.2.1 Thermoelastic damping

Thermoelastic damping occurs in materials with a high thermal conductivity. It is an internal damping mechanism that damps different frequencies at different strengths according to the following equation [11]:

$$\alpha_{\text{th}}(\omega) = \frac{\omega}{2} \eta(\omega) \approx \frac{\omega^2 R_1 C_1}{2(\omega^2 h^2 + C_1^2/h^2)} \quad (5)$$

where R_1 and C_1 are material dependent constants. In the case of the EMT140 plate reverb, these are $R_1 = 4.94 \cdot 10^{-3}$ and $C_1 = 2.98 \cdot 10^{-4}$ [10].

2.2.2 Radiation damping

Radiation damping happens when vibration is converted to acoustic energy. This happens according to the following equation [10, 11]:

$$\alpha_{\text{rad}} = \frac{1}{4\pi^2} \frac{c_a \rho_a}{\rho h} \frac{2(L_x + L_y)}{L_x L_y} \frac{c_a}{f_c} g(\psi), \quad (6)$$

$$g(\psi) = \frac{(1 - \psi^2) \ln[(1 + \psi)/(1 - \psi)] + 2\psi}{(1 - \psi^2)^{3/2}},$$

where ρ_a and c_a are the density of air and speed of sound in air respectively and $\psi = \sqrt{\frac{f}{f_c}}$. Here, f_c is the critical frequency and can be calculated using $f_c = \frac{c_a^2}{2\pi\kappa}$.

2.2.3 Damping induced by porous medium

Plate reverberators contain a porous plate positioned behind the metal plate. The distance between the two entities can be set creating a difference in low-frequency decay [10, 11]. The damping plate position changes this decay between roughly 0.6 - 5.5 seconds [5] and can be manipulated on the interface of a real plate reverb like the EMT140.

2.3 Boundary conditions

The states of the edges of a plate are referred to as the boundary conditions. In [12], the authors state that a plate can have three different boundary conditions: free, clamped or simply supported (hinged). For a rectangular plate such as the plate reverb this means that there are 27 different possible combinations of boundary conditions. Every combination will have a unique impulse response and will thus sound different. In this work, we limit ourselves to all sides being simply supported. This means that state variable $u = 0$ at $x = 0$, $x = L_x$, $y = 0$ and $y = L_y$.

3. NUMERICAL SOLUTION

The implementation approach we chose to use is the modal description (see Section 1). In this section a numerical solution of this will be presented.

The state u , as seen in the Kirchhoff-Love equation (1) can be modelled as being a summation of a number of different modes [13]:

$$u = \sum_{m=1}^M \sum_{n=1}^N q_{mn} \Phi_{mn}(x, y), \quad (7)$$

where q_{mn} is the unknown amplitude of mode (m, n) (m being the mode over the horizontal axis and n over the vertical axis of the plate) and Φ_{mn} is a modal shape defined over $x \in [0, L_x]$ and $y \in [0, L_y]$. In this model, $M \cdot N$ is the total number of modes accounted for. In theory, this is infinite. Note that – apart from the chosen time step ($1/f_s$) – the computational speed depends on M and N .

For a rectangular plate with sides L_x and L_y , using simply supported boundary conditions, $\Phi_{mn}(x, y)$ can be calculated [13]:

$$\Phi_{mn}(x, y) = \frac{4}{L_x L_y} \sin \frac{m\pi x}{L_x} \sin \frac{n\pi y}{L_y}, (m, n) \in \mathbf{Z}^+, \quad (8)$$

and q_{mn} can be found using the following update equation [13]:

$$Aq_{mn}^{t+1} = Bq_{mn}^t + Cq_{mn}^{t-1} + \frac{\Phi_{mn}(x_p, y_p)}{\rho h} P^t, \quad (9)$$

where constants A, B and C can be described in the following way:

$$A = \frac{1}{k^2} + \frac{c_{mn}}{\rho h k}, \quad B = \frac{2}{k^2} - \omega_{mn}^2, \\ C = \frac{c_{mn}}{\rho h k} - \frac{1}{k^2}.$$

Moreover, P^t is the input signal at time instance t at specified input location (x_p, y_p) , k is the chosen timestep $1/f_s$ and c_{mn} is a loss coefficient that can be set per eigenfrequency which gives a lot of control over the frequency content of the output sound.

3.1 Eigenfrequencies

The (angular) eigenfrequencies ω_{mn} can be calculated using the following equation [14]:

$$\omega_{mn} = \kappa \pi^2 \left(\frac{m^2}{L_x^2} + \frac{n^2}{L_y^2} \right). \quad (10)$$

According to [13] and experimental observation, stability of the update equation (9) can only be assured if and only if:

$$\omega_{mn} < 2f_s. \quad (11)$$

This poses a limit on the total number of modes in (7) and a creates the dependency:

$$\omega_{M(n)n}, \omega_{mN(m)} < 2f_s, \quad (12)$$

where $m \in [1, M(n)]$ and $n \in [1, N(m)]$. All the eigenfrequencies that satisfy this condition will be used in the algorithm.

3.2 Loss coefficients

In [13], the authors set loss coefficients per frequency band. In our implementation, the different damping mechanisms

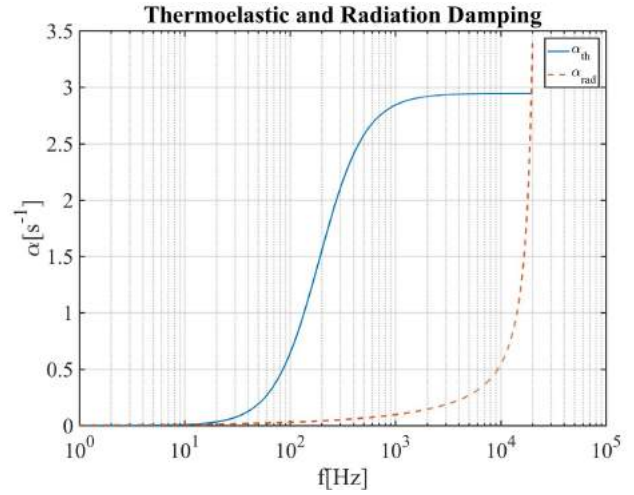


Figure 1. Thermoelastic (α_{th}) and Radiation damping (α_{rad}) for values based on the EMT140.

described in Section 2.2 have been implemented to ultimately create a loss coefficient for each individual eigenfrequency. If we set the porous medium to the furthest distance possible, the damping induced by this can be ignored. In Figure 1 the results can be seen.

The loss coefficients are then calculated in the following way [11, 13]:

$$c_{mn} = \frac{12 \ln(10)}{T_{60}}, \quad \text{where } T_{60} = \frac{3 \ln(10)}{\alpha_{tot}}, \quad (13) \\ \Rightarrow c_{mn} = 4\alpha_{tot}.$$

Here, α_{tot} is simply the addition of all of damping factors.

3.3 Dynamic outputs

The output can be retrieved at a specified point by, in a certain sense, inverting (7):

$$u_{out} = \sum_{m=1}^M \sum_{n=1}^N q_{mn} \Phi_{mn}(x_{out}, y_{out}). \quad (14)$$

The *PAI Dynamic Plate Reverb* (as described in Section 1) has the functionality of moving the input from left to right over the plate to create a flanging effect. In our implementation, not only both outputs (not inputs) are able to move, but they are able to move in elliptical and Lissajous patterns. This happens according to the following equations:

$$x_{out}(t) = R_x L_x \sin \left(S_x \cdot \frac{2\pi t}{f_s} \right) + 0.5, \quad (15)$$

$$y_{out}(t) = R_y L_y \sin \left(S_y \cdot \frac{2\pi t}{f_s} + \theta \right) + 0.5. \quad (16)$$

Here, $R_x, R_y \in [-0.5, 0.5]$ when multiplied with L_x and L_y respectively, determine the horizontal and vertical maxima of the output pattern. If one of these has a value of zero, the outputs will move in a linear shape. The shape of the pattern is determined by the horizontal and vertical speeds S_x, S_y and the phase shift θ . For example if

$S_x = S_y = 1$ and $\theta = 0.5\pi$ the outputs will follow an elliptical pattern. The patterns created for different values of S_x , S_y and θ can be seen in [15]. If either S_x or S_y has a value of zero, the outputs will again move in a linear fashion. Note, that it is perfectly possible to set different values for the aforementioned parameters for the left output channel and the right.

3.4 Changing plate dimensions

The fact that VA simulations are extremely flexible, poses a lot of interesting opportunities for model manipulation. We were interested in manipulating parameters that are physically 'fixed', such as the dimensions of a steel plate. Generally speaking, changing the dimensions of a steel plate is physically impossible to do. The challenge was thus to imagine how the sound would change if it would be possible. Looking at the variables that are dependent on the horizontal (L_x) and vertical (L_y) plate dimensions, we see that changing these would change the eigenfrequencies, the modal shapes and radiation damping. As can be seen in (10) the eigenfrequencies lower as L_x and L_y grow. Given the stability condition (11), the number of eigenfrequencies that can be accounted for ($M(n)$ and $N(m)$ in (7)) also grows as the plate dimensions grow. In the algorithm, this is implemented by adding zero values to q_{mn}^{t+1} , q_{mn}^t and q_{mn}^{t-1} (the q -vectors) in (9), where the combinations mn were non-existent in the previous update. If the plate decreases in size, the values of the q -vectors for which the combinations mn do not satisfy the stability condition anymore will be removed. Also in (7), the modal shapes change depending on L_x and L_y in (8). As long as the same number of modes is accounted for, the q -vectors will not be affected in (9) as Φ_{mn} changes. Lastly, the radiation damping α_{rad} and factors A , B and C in (9) will be updated as L_x and L_y change.

The thickness of the plate can also be changed. This changes the thermoelastic damping α_{th} in (5), the radiation damping α_{rad} in (6) and the stiffness factor κ^2 in (2) which then changes the eigenfrequencies again. It can be derived that a decrease in thickness will lower the eigenfrequencies.

4. RESULTS

In this section, the results of the implementation will be discussed. They have been informally evaluated by the authors and sound demos have been made available online¹. Unfortunately, we did not have access to an actual plate reverb, so to in order to tell whether the implementation was successful we compared it to the *PAI Dynamic Plate Reverb* and *ValhallaPlate*: already existing VA plugins. When compared to these plugins, we tried to put their settings (plate size, decay values, etc.) as close to the values of our implementation as possible. The outputs from the plugin and our implementation were then compared. The reasons for any differences in sound output we could only speculate on, as the plugins were not open-source and internal parameters were thus hidden.

¹ <http://tinyurl.com/zwsbcbl> Full link: [16]

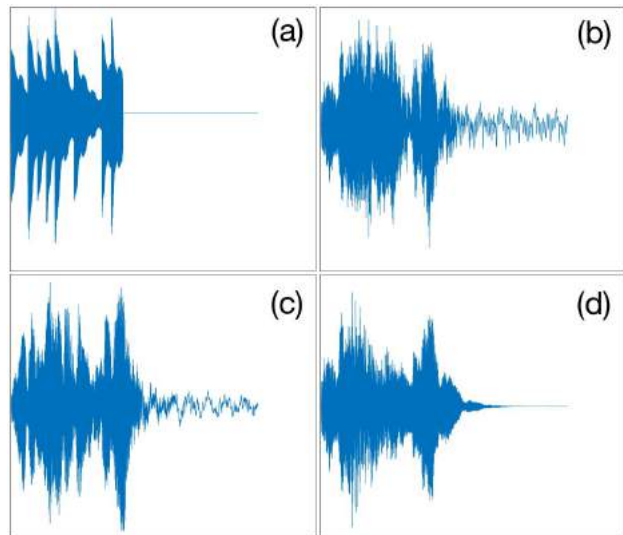


Figure 2. Plots of input and different output examples using EMT140 properties. Total length of outputs: 9.2 seconds. (a): Dry input signal, (b): Left output signal ($0.1L_x, 0.45L_y$), (c): Left moving output signal ($R_x = R_y = 0.4, S_x = 6, S_y = 5, \theta = 0.5\pi$), (d): Plate stretched from $L_y = 1 - 2\text{m}$ between 1 - 2 seconds.

As we did not have any way of comparing our novel additions (moving the outputs and changing plate dimensions) to existing solutions, we could only evaluate the output based on what we expected it to sound like. The waveforms of the input and a few output examples can be found in Figure 2.

4.1 General output

In general, the output sound is very natural, especially when combined with some dry input signal. Although no formal listening tests have been carried out, the naturalness of the has been reported by the authors. When compared to the *PAI Dynamic Plate Reverb* and *ValhallaPlate* the output has a little more low-frequency content. This is probably because the damping factors we included in our algorithm mostly attenuate the high-frequency content.

4.2 Moving outputs

Moving the outputs creates a result that sounds like a vibrato/flanging effect. This can be explained by the fact that the outputs 'travel away' or 'travel towards' sound that has already been travelling in the plate which changes the pitch of the output sound slightly.

Letting the left and right output move at different speeds or in different shapes causes the sound to arrive in the left channel and the right channel at different time instances. Perceptually, it will sound like the reverb is moving from left to right at different speeds causing a very immersive stereo effect.

4.3 Changing plate dimensions

Increasing the size of the plate creates a pitch-bend effect: lower pitch when L_x and/or L_y are increased and higher

when these are decreased. This can be explained by the fact that the eigenfrequencies are 'stretched/shortened' as the plate dimensions increase/decrease. The exact opposite happens when plate thickness is changed: if the plate gets thicker, the pitch goes up, and the other way around. This can be explained by the increase in the stiffness factor in (2) when the thickness increases.

An interesting effect occurs when input and output are combined (not 100% wet signal) as the pitch bend effect only applies to the reverb, and does not influence the dry input signal.

4.4 Dynamic loss coefficients

To the best of our knowledge, the current state of the art does not make use of loss coefficients dependent on physical parameters. We explored the possibility to make some of these parameters dynamic. The most interesting parameter we explored is the air density (ρ_a) in Equation (6). When increasing this, the sound (especially the high frequency content) will die out sooner and has a 'muffled' sound and can also be derived from the equation. Ultimately, we decided against implementing this feature, as it did not add much to the sound; it only decreased the naturalness of the plate reverb.

4.5 Computational speed

As stated in before, the computational speed depends on the total number of eigenfrequencies ($M(n)$ and $N(m)$) being accounted for in the algorithm. We found a correlation of 0.996 between this number and program speed, making decreasing the total number of eigenfrequencies our main focus. What the authors explain in [13] is the possibility of removing the eigenfrequencies that are not perceptually important. In our algorithm we propose:

$$d = (\sqrt[12]{2^{C/100}} - 1) \cdot f_c, \quad (17)$$

where C is an arbitrary amount in cents and f_c is the current eigenfrequency (in Hz) starting with the lowest eigenfrequency accounted for in the algorithm which can be calculated using $f = \frac{\omega}{2\pi}$. The algorithm will then discard the eigenfrequencies if:

$$f_i - f_c < d,$$

otherwise:

$$f_c = f_i.$$

When C is put to 0.1 cents, the total number of eigenfrequencies accounted for (using the EMT140 properties) decreases from 18,218 to 7,932, which makes a big difference in computational time. Now, our implementation only needs only needs roughly 7 seconds instead of 12 seconds to process 9.2 seconds of audio, proving that a real-time implementation would indeed be feasible. The algorithm has been tested using a MacBook Pro containing a 2,2GHz Intel Core i7 processor. The authors indeed report no significant audible difference after reducing the number of modes this way.

In order to improve computational time when the outputs are moving, $\Phi_{mn}(x_{out}, y_{out})$ is evaluated for different values of t : $t \in [1, f_s]$ with steps of $4/\max([L_x, L_y])$, before the update equation (9) and is then selected (instead of evaluated) at the appropriate times in the update equation. We found this step-size to be a good balance between speed and having minimal artefacts in the sound if the speed is not set to be too high.

Changing the plate dimensions greatly influences the computational time as the eigenfrequencies need to be recalculated many times. To improve computational time, the eigenfrequencies and the modal shapes are reevaluated every 100th time-step, according to the current (new) dimensions of the plate. As with the previous chosen time-step, found this to be a good balance between speed and having minimal artefacts.

5. CONCLUSION AND FUTURE WORK

In this paper we presented a VA simulation of plate reverberation. The output of the implementation sounds natural and parameters like in- and output positions and plate dimensions have been made dynamic resulting in a very unique and interesting flanging and pitch-bend effect (respectively), something which has not yet been achieved by the current state of the art. Another novelty is that we included thermoelastic and radiation damping that influence every eigenfrequency independently.

In the future we would like to create a real-time plugin containing all that is presented in this paper. In order to do so, the algorithm needs to be optimised, especially computationally heavy processes like moving the pickups and changing the dimensions of the plate. When this is achieved, the plugin will be tested with musicians in order to test usability and whether the output sound is satisfactory. Furthermore, we would like to add damping induced by a porous medium to our implementation. Even though it has been ignored in this work, it is an important feature in the EMT140. Lastly, we would like to explore the possibilities of changing other parameters of the plate using the presented model as a basis. The shape and the structure of the plate, for example, would be very interesting to make dynamic. Moreover, different materials, such as gold and aluminium, or even non-metallic materials like glass, could be explored and result in some interesting timbres.

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